Withdrawal of Flat Plates from Power Law Fluids

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A new one-term withdrawal expression for power law fluids is developed from theoretical equations. Comparison with film thickness data for three carbopol fluids of $\alpha = 1.79$ to 2.68 indicates agreement similar to that observed with valid Newtonian theories. The agreement was verified by comparison with independent measurements of entrainment flow rate.

Entrainment in continuous, steady state vertical withdrawal of flat sheets from non-Newtonian liquids has several applications, some of which were recently reviewed (*11*). To date, however, no theory has successfully described the magnitude of entrainment (film thickness h_0 or flow rate w) observed experimentally with pseudoplastic fluids. We are concerned here with fluids described by the power law model

$$-\frac{dv}{dy} \equiv [a_1{}^{\alpha}|T_{yx}|^{\alpha-1}]T_{yx} \tag{1}$$

The primary purpose of this paper is to develop a theoretical expression for power law fluids which describes the magnitude of entrainment and the influence of withdrawal speed u and other parameters. The new expression is developed by patching two solutions.

BASIC EQUATIONS FOR NON-NEWTONIAN FLUIDS

The curvature match method used by Landau-Levich (*7*) and others (*8, 11*) to derive withdrawal theories is based on matching surface curvatures in the meniscus region as follows. The withdrawal film is first described by three regions: upper (region 1 or constant thickness), middle (region 2 or dynamic meniscus), and lower (region 3 or static meniscus). The equation of capillary statics is taken to apply to the lower portion of the withdrawal meniscus, region 3. Curvature profiles in regions 2 and 3 are then derived from momentum equations, locations for estimating curvatures are selected, and the resulting curvatures matched.

Where the vertical coordinate is x and meniscus thickness is h, the meniscus equations have been written in nondimensional form by use of h_0, the film thickness in in the constant thickness region. Thus, the nondimensional parameters are distance ($R = x/h_0$) and meniscus thickness ($L = h/h_0$). The curvatures ($L'' \equiv d^2L/dR^2$) of the static and dynamic regions (L''_S and L''_D, respectively) are evaluated at the following limiting conditions: $L''_{SL} \equiv \lim_{R\to 0} L''_S$ and $L''_{DL} \equiv \lim_{R\to\infty} L''_D$.

The last step in the development of withdrawal theories by the curvature match method is the rearrangement of the function described by the following match of curvatures, $L''_{SL} = L''_{DL}$.

The first basic equation (for both Newtonian and non-Newtonian fluids) is that for the static meniscus region; the resulting limiting curvature is $(2\rho g/\sigma)^{1/2}$. In terms of the nondimensional film thickness D_0, the limiting curvature is given by

$$L''_{SL} = D_0\sqrt{2} \tag{2}$$

The second basic equation is the dynamic meniscus equation used to evaluate L''_{DL}. The dynamic meniscus equation incorporates the region 1 momentum equation results into the one-dimensional, incompressible, isothermal momentum equation for region 2. The latter contains terms for viscous shear, gravity, and surface tension. The dynamic meniscus equation for power law fluids has been shown to be (*4, 8*)

$$\frac{L^{\alpha+2}}{(\alpha+2)}\left[T_1^{\frac{(\alpha+1)}{\alpha}} + \frac{L'''}{C_1}\right]^{\alpha} = (L-1) + \frac{T_1{}^{\alpha+1}}{(\alpha+2)} \tag{3}$$

with

$$L = 1,\ L' = 0,\ L'' = 0 \text{ at } R = 0. \tag{4}$$

The nondimensional parameters T_1, C_1, and α are constants in Equation (3). Parameters T_1 and C_1 are defined in terms of film thickness, speed, and fluid properties.

The Ellis fluid is described by the low shear parameter a_0, as well as by a_1 and α:

$$-\frac{dv}{dy} \equiv [a_0 + a_1{}^{\alpha}|T_{yx}|^{\alpha-1}]\,T_{yx} \tag{5}$$

The dynamic meniscus equation for an Ellis fluid was derived similarly (*8*). Where $T_0 = h_0\,(a_0\rho g/u)^{1/2}$ and $C_0 = (u/a_0\sigma)$, it is given by

$$\frac{L^3L'''}{3C_0} + \frac{L^{\alpha+2}}{(\alpha+2)}\left[T_1^{\frac{(\alpha+1)}{\alpha}} + \frac{L'''}{C_1}\right]^{\alpha} = (L-1) - \frac{T_0{}^2\,(L^3-1)}{3} + \frac{T_1{}^{\alpha+1}}{(\alpha+2)} \tag{6}$$

Power law Equation (3) is a special case of Equation (6) for $a_0 = 0$.

APPROXIMATIONS IN THE MATCH METHOD

It is useful to divide the approximations implied in the curvature match method into two groups and name them as (a) the basic assumptions and (b) the special assumptions. The seven basic approximations are:

1. Constant thickness (region 1).
2. Negligible viscous force (region 3).
3. Estimated curvature location (region 3).
4. Estimated curvature location (region 2).
5. One-dimensional flow (region 2).
6. Negligible flow effects on interfacial momentum jump. (Use of La Place equation and constant surface tension) (region 2).
7. Negligible film thickness gradient ($dh/dx < 1$) (region 2).

The first four basic assumptions are general approximations of the method. The last three are used in the development of the dynamic meniscus equation.

The basic assumptions have been shown experimentally to be valid for withdrawal over a wide range of speeds and fluid properties. Constant thickness in region 1 has been verified by direct measurement of film thickness vs. height (*4, 11*). The other assumptions have been verified by comparison of data (measured film thicknesses and flow rates) with theory (values predicted by using the meniscus match method). The assumptions have been discussed by Levich (*7*) and others (*14*).

The special approximations are those made in solving the dynamic meniscus equation for L''_{DL} and at present are considered to be the controlling limitations on the match method. Because the special assumptions vary from case to case, they are presented below. Two examples (*12*) of the special approximations are a negligible gravity term (negligible T_0 or T_1) and a thin meniscus ($L < 2$).

THE SPECIAL CASE OF NEWTONIAN FLUIDS ($\alpha = 1$)

For a Newtonian fluid $a_1 = 1/\mu$ and the T_1, C_1 parameters become $T_0 = h_0(\rho g/\mu u)^{1/2}$ and $C_0 = \mu u/\sigma$. Meniscus Equation (3) reduces to the Landau-Levich (*7*) equation

$$\frac{L^3}{3}\left[T_0^2 + \frac{L'''}{C_0}\right] = (L-1) + \frac{T_0^2}{3} \qquad (7)$$

The special assumption of a thin meniscus ($L < 2$) has been used (*12*) to simplify Equation (7) to

$$L''' = \frac{(L-1)}{L^3}(3\,C_0)\,(1 - T_0^2)$$

$$= \frac{(L-1)}{L^3}(3)\,(C_0 - D_0^2) \qquad (8)$$

Equation (8) implies that $L''_{DL} = 0.642(3)^{2/3}(C_0 - D_0^2)^{2/3}$, without any further approximations. The two-term theory for Newtonian fluids, developed (*12*) by matching the L''_{DL} and L''_{SL} curvatures cited above, is given by the following prediction of film thickness (*5*):

$$u = \left[1.09\left(\frac{\sigma}{\rho g}\right)^{1/4}\left(\frac{\rho g}{\mu}\right)\right] h_0^{3/2} + \left(\frac{\rho g}{\mu}\right) h_0^2 \quad (9)$$

Theoretical Equation (9) is compared with continuous withdrawal data (*2, 4*) in Figure 1. Agreement in magnitude of thickness (Figure 1), in influence of viscosity (Figure 1), and in influence of speed (*4*) shows that the curvature match method is valid for Newtonian fluids. Equation (9) has been shown to be valid for at least a

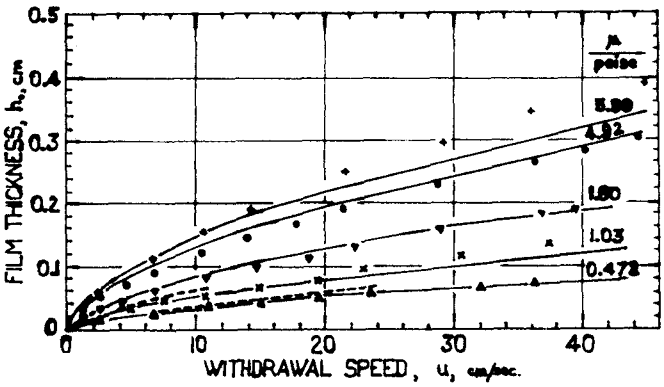

Fig. 1. Verification of Newtonian theory [Equation (9)].

10,000 fold range of nondimensional speed, C_0 (*12*).

Because the numerical value of 0.642 is important in this paper, we explain how it was obtained. First, Equation (8) was replaced by using a change in the R variable, by the resultant equivalent form is

$$L''' = (L-1)/L^3 \qquad (10)$$

Because L''' vanishes as R and L approach infinity [see Equations (8) and (10)], L'' asymptotically approaches a numerical constant at large R and L. We call the numerical value the *curvature coefficient* P and define P as follows:

$$P \equiv \lim_{L\to\infty} L'' \qquad (11)$$

The P value for Equation (10) has been found by numerical integration to be 0.642.... (*1, 7, 12*). The limiting value of dynamic curvature is thus obtained as $L''_{LD} = P\,(\phi)^{2/3}$, where ϕ is based on the change of variable.

At low speeds and low thicknesses, the right-hand term in Equation (9) becomes negligible, and Equation (9) reduces to the following one-term theory:

$$u = 1.09\left(\frac{\sigma}{\rho g}\right)^{1/4}\left(\frac{\rho g}{\mu}\right) h_0^{3/2} \qquad (12)$$

Equation (12) was derived in 1942 by Landau and Levich (*7*) using the special assumption of negligible T_0 in Equation (8). It was first verified in 1945 by Deryaguin and Titiyevskaya (*1*) using removal experiments.

PREVIOUS POWER LAW THEORY (ONE TERM)

Consider two parameters P_L and k. Here P_L is a curvature coefficient which is a function of alpha only, as shown in Figure 2. The parameter k is a function of fluid properties as follows:

$$k \equiv \left[\frac{(2+\alpha)(\rho g/\sigma)^{\alpha/4}}{(3)^\alpha (1.09)^\alpha (a_1 \rho g)^\alpha}\right]^{\frac{2}{(2+\alpha)}} \qquad (13)$$

The first theoretical prediction of h_0 for power law fluids (*4*) is a one-term theory. It may be written concisely in terms of the fluid property parameters k and P_L as

$$h_0 = \left[\frac{P_L}{0.642}\right]^{\frac{3\alpha}{(2+\alpha)}} k\; u^{\frac{2}{(2+\alpha)}} \qquad (14)$$

The one-term theory properly predicts the observed functional dependence of entrainment on speed (*4*). However, the predicted magnitude of entrainment is substantially larger than that observed experimentally, as shown in Figure 3.

The data shown in Figure 3 were taken with the same apparatus used for Figure 1 data. The liquids used were several aqueous carbopol solutions which have been shown to follow power law Equation (1) and have exhibited no measurable yield stress (2 to 4). Elastic fluids are beyond the scope of this work.

Equation (14) was developed by using the special assumption of negligible T_1 in Equation (3) and by changing variables. The resultant meniscus equation was Equation (15), from which L'' and $P_L(\alpha)$ were obtained, as shown in Figure 4 for two values of alpha:

$$L''' = \left[\frac{L-1}{L^{(\alpha+2)}} \right]^{\frac{1}{\alpha}} \tag{15}$$

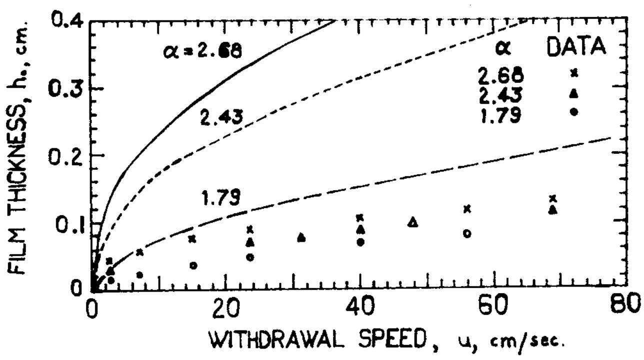

Fig. 3. Comparison of data with the first one-term theory [Equation (14)].

PREVIOUS POWER LAW THEORY (TWO TERM)

A more recent power law theory (8) may be written (5) in terms of fluid property parameters n and m as

$$u = \left(\frac{0.642}{P_N} \right)^{3/2} mnh_0^{\alpha+0.5} + nh_0^{\alpha+1} \tag{16}$$

where

$$m = 1.09 \left(\frac{3\alpha}{2+\alpha} \right) \left(\frac{\sigma}{\rho g} \right)^{1/4} \tag{16a}$$

and

$$n = (\alpha_1 \rho g)^\alpha \tag{16b}$$

The derivation of Equation (16) indicated that the relevant curvature coefficient $P_N(\alpha)$ is constant for all α, as shown in Figure 2. Thus

$$P_N(\alpha) = 0.642 \tag{17}$$

The two-term theory of Equation (16), when compared with the data of Figure 4, was found (6) to predict the magnitude of thickness more closely than the one-term theory [Equation (14)]. However, the predicted effect of speed differs from data.

To discuss the speed data, we define the functional dependence of film thickness on speed as the derivative d on a log-log plot. Thus, for any given fluid

$$d \equiv \frac{\partial \ln h_0}{\partial \ln u} \tag{18}$$

Hereafter we refer to d as the slope. The slope is constant for one-term theories but is a function of speed for two-term theories.

Table 1 shows that two-term Equation (16) is not satisfactory for predicting the observed speed effect; differences in slopes are in the range of 20 to 35%. Table 1 also shows that the maximum possible slopes of the theory were less than those observed experimentally. Average values are reported because both the theoretical and observed slopes were nearly constant in the speed range studied.

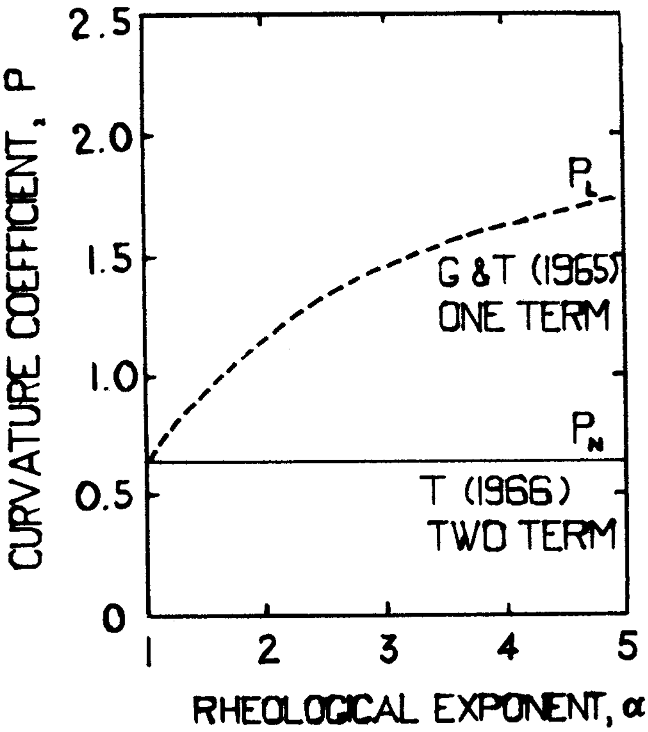

Fig. 2. Two theoretical predictions of curvature coefficients; ------ P_L of the one-term theory [Equation (14)], ——— P_N of the two-term theory [Equation (16)].

This two-term theory was derived from Ellis meniscus Equation (6) by using the special assumptions expressed by the inequality Equation (19) and the thin meniscus ($L < 2$):

$$T_1^{\frac{\alpha+1}{\alpha}} > \frac{L'''}{C_1} \tag{19}$$

By using these assumptions, Equation (6) was simplified by using the binomial theorem, truncating terms, and by

TABLE 1. COMPARISON OF SPEED EFFECT SLOPES*, TWO-TERM THEORIES†

Experimental conditions			
Fluid α	1.79	2.43	2.68
Wt. % carbopol, aqueous	0.16	0.18	0.19
Speed range, u (cm./sec.)	0.3 to 80	0.7 to 80	0.3 to 80
Observed slope			
Data d	0.51	0.46	0.44
Theory slope (range)			
First term only, d	0.44	0.34	0.31
Second term only, d	0.36	0.29	0.27
Average slope (expt. speeds)			
P_N theory, d [Equation (16)]	0.42	0.34	0.31
P_V theory, d [Equation (22)]	0.40	0.31	0.28

* Slope d is defined by Equation (18).
† Equations (16) and (22).

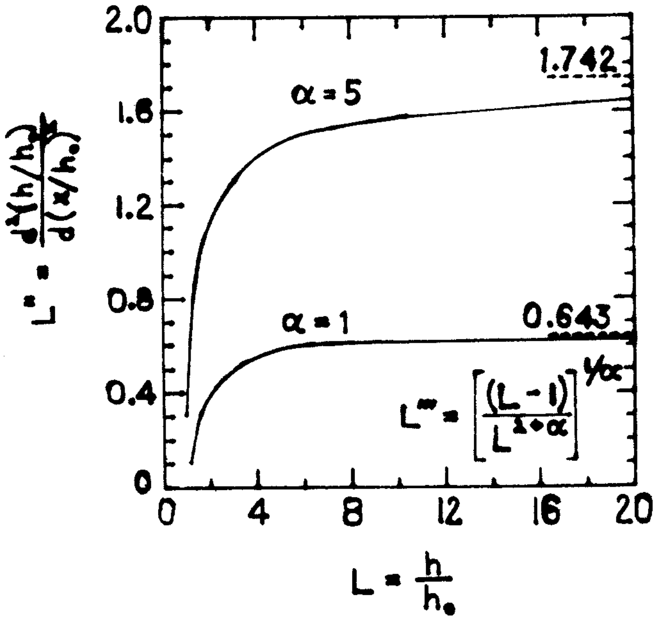

Fig. 4. Effect of α on asymptotic L'' [Equation (15)].

changing the R variable. The result was Equation (10). Equation (16) was determined from the resultant Ellis Theory by letting $a_0 = 0$.

PRESENTATION OF THE NEW EXPRESSION

The one-term theory predicts the proper functional dependence of speed and fluid properties but apparently involves an incorrect curvature coefficient. The two-term theory gives a better prediction of magnitude and thickness but an improper speed effect. Consideration of these two power law theories suggests a patch method for obtaining an expression from theoretical values. We use the curvature coefficient from the two-term theory [P_N of Equation (17)] together with the influence of speed and properties from the one-term theory [Equation (14)]. Patching these two theoretical expressions leads to the new expression, where k is given by Equation (13):

$$h_0 = k(u)^{\overline{\frac{2}{(2+\alpha)}}} = k(u)^d \qquad (20)$$

The reasons for choosing the patch method include belief that (a) the dynamic meniscus Equation (6) is applicable to pseudoplastic fluids (inelastic, non-Newtonian), (b) the special case solutions of Equation (6) offer approximate descriptions of withdrawal, (c) the convergence rate for L'' should be as rapid for pseudoplastic fluids as for Newtonian fluids, and (d) reasonable P for pseudoplastics are no greater than $P_N(\alpha)$. These four reasons are discussed below.

DISCUSSION OF BASIC AND SPECIAL APPROXIMATIONS

Consider the following. The seven basic assumptions have been verified as a sound theoretical basis for Newtonian fluids (*12* to *14*) over a wide range of conditions. There are no known reasons to suspect that the basic assumptions are not applicable to proper rheological models. The dynamic meniscus equation contains all influences of flow and rheological properties present in the curvature-match-method description. The special approximations of the meniscus equation have been shown, for Newtonian plates (*12*) and cylinders (*13, 14*), to contain the controlling limitations.

Therefore, it appears that any disagreement between approximate solutions of Equation (6) and data is due to the special assumptions made. Further consideration led to the reasons (a) and (b) listed above.

Special case solutions (of any extension of the dynamic meniscus equation) require testing to determine regions of speed and properties for which the solutions are valid. Such extensions and verification tests have been presented for plates (*12*) and cylinders (*13, 14*) for Newtonian fluids. The extension of the dynamic meniscus equation to Ellis fluids is given by Equation (6); four special solutions for power law fluids (*4, 8, 9*) and two for Ellis fluids (*6, 8*) are available. Tests on these special solutions indicate that no region of validity has been found to date (*4, 6, 9*) and that deviations for power law fluids are not due primarily to lack of a low shear parameter (*6*). The lack of success with the special solutions suggested a study of curvature coefficients.

The first two-term theory for power law fluids has very large curvature coefficients ($P_V > P_L$ for all $\alpha > 1$) (*4*). The theory may be expressed as (*9*)

$$u = \left(\frac{0.642}{P_V}\right)^{3/2} m\, n\, h_0^{\alpha+0.5} + n\, h_0^{\alpha+1} \qquad (21)$$

Comparison of two-term Equations (16) and (21) showed the clear superiority of the P_N theory in predicting magnitude of entrainment. This indicated that deviation of early theories were due largely to high P values and to slow L'' convergent rate rates. The study of P values led to

TABLE 2. DEVIATION BETWEEN FILM THICKNESS DATA (2) AND EQUATION (20)

Experimental conditions			
Fluid α	1.79	2.43	2.68
Wt. % carbopol, aqueous	0.16	0.18	0.19
Speed range, u (cm./sec.)	0.3 to 80	0.7 to 80	0.3 to 80
Number of data points (2)	9	8	10
Equation (20), k	0.01057	0.01779	0.02265
Slope comparison, d			
Equation (20)	0.53	0.45	0.43
Data	0.51	0.46	0.44
Magnitude deviations, h_0			
Mean deviation, %*	16%	4%	6%
Extreme deviations, %*			
Negative	−17%	−1%	−19%
Positive	+32%	+8%	+7%
Smallest	9%	1%	1%

* Relative deviations, $(h_T - h_D)/h_D$, where T indicates Equation (20) and D data.

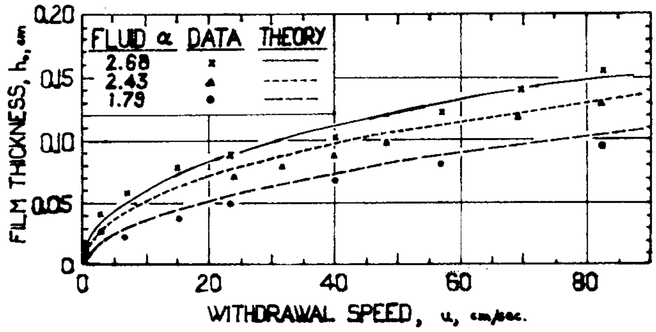

Fig. 5. Verification of the power law Equation (20) (film thickness).

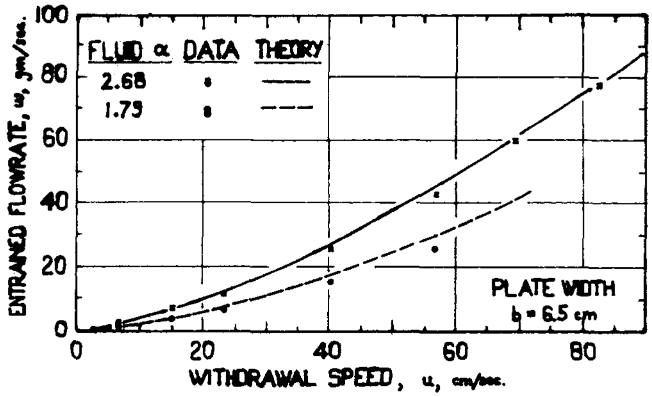

Fig. 6. Verification of the power law Equation (20) (mass flow rate).

reasons (c) and (d) listed above and to the patch method and Equation (20).

The main reason for presenting Equation (20) is the basic one used for any theoretical expression, it describes nature.

VERIFICATION OF EQUATON (20)

Table 2 shows that Equation (20) properly predicts the dependence on speed. The average deviation of predicted slopes for the three fluids is about 2 to 4%, which is within experimental error.

Figure 5 shows that Equation (20) also properly predicts the magnitude of film thickness. The data lie above and below the predictions in a manner characteristic of experimental scatter, such as in Figure 1. Agreement in thickness was good at all speeds studied. As indicated by Table 2, the overall average of the relative deviations is about 9%. Because of the large disagreement noted at lower speeds for the four-term Ellis theory (*6*) and all two-term theories (*4, 6, 9*), the agreement of Equation (20) at speeds below 20 cm./sec. is specially noteworthy.

Another test of withdrawal predictions is measurement of the mass flow rate w which is entrained on a belt of width b. The flow rate for a power law fluid is related to the laminar thickness (*4*) by

$$w = \rho b h_0 u \left[1 - \frac{(a_1 \rho g)^\alpha h_0^{\alpha+1}}{(\alpha+2)u} \right] \tag{22}$$

The Figure 6 comparison of independent w measurements (*2, 4*) with predictions from Equations (20) and (22) shows agreement in both magnitude and functional dependence. Therefore, Figure 6 provides an independent verification of Equation (20).

Withdrawal data are also available (*2, 4*) for a fluid, 0.20 wt. % carbopol, having $\alpha = 3.25$. Equation (20) properly describes the slope of this high α fluid (d values of 0.38 predicted vs. 0.39 date), but predicted values of film thickness do not agree numerically with data.

The high α result is not understood, but it may indicate that some small yield stress is present; yield stress was noted with a 0.40% solution (*4*). Until other evidence is available, however, we simply note that Equation (20) may or may not apply for alpha values above 3.

We conclude that Equation (20) is the first theoretical expression for predicting withdrawal entrainment for pseudoplastic liquids. Because the Newtonian case of $\alpha = 1$ [Equation (12)] has been verified with data (*12*), Equation (20) is considered verified over the α range of 1 to 2.7. Equation (20) is believed to be subject to upper speed limit restrictions, such as that found with Newtonian fluids, but the speed limit is known only for $\alpha = 1$.

DISCUSSION OF COEFFICIENTS

A semiempirical expression for power law fluids has been reported (*4, 10*). It was developed for a limited alpha range by using an entirely empirical method (*4*) to describe P and the magnitude of film thickness; the influence of α on P was given (*10*) as $0.41 + (0.25/\alpha)$. Although the average deviation of the semiempirical expression from data appears to be about the same as that of Equation (20), the latter is preferred because of the theoretical source of the coefficient.

Another possible one-term expression is formed by selecting the α decreasing P_S curvature coefficient (9) for patching with Equation (14). However, the patched P_S expression does not properly predict the magnitude of film thickness. For the Figure 5 range of speed and α, for example, the P_S expression indicated h_0 values which were almost independent of α; film thickness values predicted at 10 and 80 cm./sec. for the three α were within ± 10% of 0.0020 and ± 15% of 0.055 cm., respectively.

THE D FORMS

The new expression for h_0 may be placed in nondimensional, speed-explicit form comparable to Ellis and Newtonian fluid forms (*5*) by using a_0 as a dummy variable as follows:

$$C_0 = \frac{u}{a_0 \sigma} = \frac{(h_0/k)^{1+(\alpha/2)}}{a_0 \sigma} \tag{20a}$$

Equation (20*a*) can also be placed in the comparable D_0 form (*5*) by use of the fluid property parameter B:

$$C_0 = \frac{(1.09)^\alpha (3)^\alpha B}{(2+\alpha)} D_0^{1+(\alpha/2)} \tag{20b}$$

The D_0 form that avoids a_0 involves $A = \sigma a_1^\alpha (\rho g \sigma)^{(\alpha-1)/2}$; Equation (20) becomes

$$u = \frac{(1.09)^\alpha (3)^\alpha A}{(2+\alpha)} D_0^{1+(\alpha/2)} \tag{20c}$$

CONCLUSIONS

The new, one-term power law expression of Equation (20) has been verified over the alpha range of 1 to 2.7 by film thickness data and by flow rate data. It is the first valid withdrawal expression for inelastic, pseudoplastic materials based entirely on theoretical values.

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NOTATION

a_0 = rheological parameter, 1/poise, Equation (5)
a_1 = rheological parameter, Equation (1)
B = nondimensional parameter, $[a_1(\rho g \sigma)^{1/2}]^\alpha / [a_0 (\rho g \sigma)^{1/2}]$
b = width of flat plate, cm.
C_0 = nondimensional speed, $u/a_0\sigma$
C_1 = nondimensional parameter, $(h_0^{\alpha-1}u)^{1/\alpha}/(a_1\sigma)$
D_0 = nondimensional thickness, $h_0(\rho g/\sigma)^{1/2}$
d = slope, the derivative $\partial \ln h_0 / \partial \ln u$
h = meniscus thickness, cm.

h_0 = film thickness, constant thickness region, cm.
k = fluid property coefficient, Equation (13)
L = nondimensional meniscus thickness, h/h_0
L'' = d^2L/dR^2
L''_{DL} = limiting value, dynamic region 2
L''_{SL} = limiting value, static region 3
n = fluid property coefficient, $(a_1 \rho g)^\alpha$
m = fluid property coefficient, Equation (16a)
P = curvature coefficient, nondimensional, Equation (11)
P_L = large P value ($P_L > P_N$), Figure 2
P_N = Newtonian P value, constant, Figure 2
P_S = small P value ($P_S < P_N$)
P_V = very large P value ($P_V > P_L$), Equation (22)
R = nondimensional height, x/h_0
T_0 = nondimensional parameter, $h_0(a_0 \rho g/u)^{1/2}$
T_1 = nondimensional parameter, $h_0[(a_1 \rho g)^\alpha/u]^{1/(\alpha+1)}$
u = withdrawal speed, cm./sec.
v = local velocity, cm./sec.
w = mass flow rate, g./sec.
x = height in meniscus, cm.
y = coordinate

Greek Letters

α = rheological exponent, nondimensional, Equation (1)
μ = Newtonian viscosity, poise
ρ = density, g./cc.
σ = surface tension, dyne/cm.
τ_{yx} = shear stress, dynes/sq.cm.

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Concentration Fluctuations and Chemical Conversion Associated with Mixing in Some Turbulent Flows

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Microconductivity probes were used for the measurement of point values of mean concentrations and root-mean-square concentration fluctuations for mixing of salt solutions in turbulent shear flows. Mixing studies covered a range of flow conditions, with Reynolds numbers on the order of 10^4, and included ducted turbulent jets, dispersion in turbulent pipe flow, a plane mixing zone, and several multiple injection systems. The test sections all had characteristic dimensions of about 2 cm. The results for mixing experiments are compared with available previous work. Lack of resolution for root-mean-square concentration fluctuations in some previous work is indicated.

Reaction product concentrations for a rapid, second-order, irreversible reaction in multiple injection systems, the plane jet, and the mixing zone were also obtained. The relation between conversion in reaction experiments and fluctuation level and decay in the equivalent mixing experiments is illustrated.

The study of mass transfer in turbulent flows has, for a long time, implied the measurement of mean concentration profiles and perhaps calculation of eddy diffusion coefficients from these results. However, the concentration measured at a point in a turbulent flow in general exhibits essentially random fluctuations with various frequencies and amplitudes about the mean value. The second moment or variance of the concentration fluctuations is therefore the next bit of information needed for a more detailed look at turbulent mass transfer. In fact, the standard deviation of the concentration fluctuations, known as the *intensity*, may be of the same order of magnitude as the mean concentration.

The fluctuation intensity is particularly useful since, even with uniform mean concentration in the field, mixing

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